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THE MONIST

HERMANN GRASSMANN.

1809-1877.

WE like to believe that the final significance of any thinker's work is independent of his time and place and is fixed by reference to some absolute standard. However that may be, it seems quite clear that his importance in his own age, and hence his effect on the next succeeding generations, depends to some extent on other factors than his intrinsic value. And so in judging that value we must distinguish plainly between it and what we might call the relative or historical importance of the man's work. This latter may well be compared to the potential of a body in electrostatics. For just as that potential depends not only on the actual charge on the body, but also on the charges on neighboring bodies; so also the relative importance of a man is not determined alone by the content of his life and work, but is affected also by his *milieu* and by the reactions of that *milieu* to it.

This is the reason why the contemporary estimate of a thinker is often so utterly wrong. At the time, the external man and his work are more easily seen; but the subtle tendencies of the age are not so readily understood, nor can the observer escape the distortion of vision wrought by prevailing influences on himself. So it comes about that he who is written down a failure in one age may stand out a very genius in the next.

These reflections are quite pertinent to any inquiry into

the life and work of the author of the *Ausdehnungslehre*—Hermann Günther Grassmann, the distinguished mathematician whose own generation passed him by. Although he reached eminence in other branches of human activity, we speak of him as a mathematician because that was certainly the subject he loved most and in which his influence will be most felt in the future. Of him, on the occasion of his centenary (1909), F. Engel¹ could say: "To-day he is known by name to mathematicians, but few have read his writings. Even where his ideas and methods have been diffused in mathematical physics people learn them second-hand, sometimes not even under his name." So in Grassmann we have a straightforward example of a man between whose work and whose influence on his own and immediately succeeding generations we must sharply distinguish if we are to avoid underrating his significance.

He was born on April 15, 1809, in Stettin.² His father, Justus Günther Grassmann, was a teacher in the Gymnasium there, and was himself a good mathematician and physicist.³ His school days passed without his showing any inclination or aptitude in special studies. He had however great skill in and fondness for music, and received a good foundation in piano and counterpoint from the famous composer Loewe. The latter was appointed teacher in the Stettin Gymnasium in 1820 and lived for the first year in the house of the Grassmanns, where he found very congenial society in Hermann and his brothers and sisters, all of whom were musical. With them Loewe often used to try over his new quartettes.

Of Grassman's inner development during these out-

¹ F. Engel, Speech on "Grassmann in Berlin," to the Berliner Mathematische Gesellschaft (1909). To this I owe most of the information about Grassmann's early life given in what follows.

² The same date as Euler.

³ He invented an air-pump cock which was given his name, and also constructed a useful index notation of crystals.

wardly calm and uneventful years we can form a clear picture from his own writings. For in 1831 he wrote an account of his life in Latin in connection with the examination for his teacher's certificate; and later, in 1834, he handed in an autobiography to the Konsistorium in Stettin when he was passing his first theological examination. He refers to those earlier years as a period of slumber, his life being filled for the most part with idle reveries in which he himself occupied the central place. He says that he seemed incapable of mental application, and mentions especially his weakness of memory. He relates that his father used to say he would be contented if his son Hermann would be a gardener or artisan of some kind, provided he took up work he was fitted for and that he pursued it with honor and advantage to his fellow men. As he usually spent his holidays in the country among relatives, and nearly always in the families of clergymen, he conceived the desire to prepare himself for the ministry. But he soon came, partly from the ridicule of his companions and partly from the warnings of his parents, to doubt his capacity. He says however that, after his course of instruction for confirmation, a light came into his dreams. Suddenly he determined to exercise all his intellectual powers and to overcome as far as possible the phlegmatic character of his temperament. And this resolution he carried out with resistless energy.

F. Engel⁴ sums up these early years in the following words: "He does not belong to those early ripening geniuses who, even in childhood's years, know whither their gifts will lead them, and turn without doubt or hesitation to that branch of knowledge to which they are called. He was exceptionally gifted on too many sides for that. But even these many-sided gifts by no means showed themselves at the beginning; and that they developed themselves richly

⁴ *Ibid.*

later came by no means without effort, but was the direct result of many years of concentrated work which he did in order to develop his character and to solidify his moral outlook and grasp of life."

In the August of 1827 Grassmann and his elder brother Gustav entered the University of Berlin with the intention of studying theology. Two days after their arrival Hermann wrote a droll letter to his mother vividly describing how they had settled in. He tells how they had to climb seventy-two steps to their attic dwelling at 53 Dorotheenstrasse at the corner of Friedrichstrasse. They had only room for their beds and two chairs, but he comments humorously on their extra fine look-out over the gardens and houses of the city, and adds that though the rooms were small they could be the more easily heated. His landlady will be recognized by students all the world over in his pen picture, "If she does talk too much, she is very pleasant and industrious." Particularly amusing is the manner in which he tells how they had spent practically all their money in two days. He enumerates all the possible and impossible things on which they had *not* spent the money, and finally confesses that their sudden impecuniosity was due to the piano which they luckily bought for 50 Taler.

Grassmann admitted later that when he first came to the university he was quite dependent on the guidance of the professors. He was easily impressed by the lectures he heard and tended to fit in his studies with the lectures he chanced upon rather than to take those corresponding to a course of study. At first he came specially under the influence of the well-known church historian Neander. Gradually, however, he became attracted more and more to Schleiermacher to whom he acknowledges great indebtedness. He wrote: "Early in my second year I attended Schleiermacher's lectures, which of course I did not understand; but his sermons began to exercise an influence upon

me. However it was not until my third, and last, year that Schleiermacher entirely engaged my thought, and although at that time I was more occupied with philology, yet I then for the first time recognized how one could learn something from him for every branch of knowledge, because he aimed less at giving positive information than at making us capable of attacking each investigation in the right way and of carrying it on independently." From this we can see how Grassmann was coming to feel the joy of original creative work.

Though he had studied theology with his heart in his subject, he had by this time reached the decision to lay it aside. He says that he had noticed that clergymen who lived in country parishes, shut off from intercourse with scholars, lost grasp of their studies, however enthusiastic they had previously been, and ceased to pursue any investigation on their own account. To escape such a fate he decided to prepare himself as broadly as possible. For this reason he began the study of philology, but he continued it from sheer love of the subject. He had also made the discovery by this time that academic lectures are only of profit if taken in moderation; so he confined himself to two courses under Professor Boeckh, on the history of Greek literature and on Greek antiquities respectively. But he planned out a tremendous course of study, intending to begin with Greek grammar, then to read the Attic authors chiefly the historians, with the study of whom he would combine Greek history and antiquities—next the tragedians with mythology and poetic forms, and afterward Homer and Herodotus. Meanwhile he would seek variety by reading Roman authors. Finally, as he intended to follow his linguistic studies with mathematics, he meant to save Plato and Demosthenes until he began that study.

This exhaustive program he was not able to complete in Berlin. When he had reached the Attic authors he was

taken ill in consequence of over-work. He describes his illness as neither severe nor dangerous, but it compelled him to slow down and to introduce more variety in order to avoid mental strain.

In this way he was led to the study of the sciences, but he showed his growing independence by working free of the schools. He did not attend a single mathematical lecture while a student in Berlin.

We may now see how wide his range of interests was throughout his university career. He seems to have been striving for as broad a foundation as possible, while at the same time he was building up a truly scientific attitude of mind which would enable him successfully to attack any subject he might turn his attention to. It is as though, as Engel says, he knew from the first that it would be necessary in his life to have more than one iron in the fire.

In the autumn of 1830 he returned to Stettin, and late in the following year took an examination for a teacher's certificate before the Scientific Examination Commission in Berlin. It was at this examination that he handed in the Latin autobiography we have previously referred to, and concerning which Köpke, rector of the monastary school of the Grey Friars in Berlin, comments "*Specimen tum propter rerum ubertatem tum propter stili venustatem et elegantiam laude dignum.*" He was given permission to teach philology, history, mathematics, German and religious knowledge in lower and middle classes; but the commission at the same time expressed their expectation that he might easily perfect himself for teaching ancient languages and mathematics in all classes. This may have stimulated Grassmann to further mathematical studies, though he had already thrown himself with energy into them under the influence of his father, whose text-books he would naturally use.

He became assistant teacher (*Hilfslehrer*) in the Stettin gymnasium, and in 1832 began to lay the foundations of his great work, the "Theory of Extension" (*Ausdehnungslehre*). He began by working at the geometrical addition of straight lines, or what we now call vector addition. From this he was led to the notion of the geometrical product of straight lines. The direct influence of his father can best be shown by his own words:⁵ "But I had not the slightest idea into what a rich and fruitful province I had here arrived; rather did this result⁶ appear to me to be little worthy of notice until I combined it with a closely related idea. Namely, by following it up with the same idea of the product in geometry as my father had held,⁷ it became evident to me that not only the rectangle but also the parallelogram in general may be considered as the product of two adjoining sides."

He goes on to add that he was surprised to find that he had thus reached a product which changed in sign if its factors were interchanged. And this, together with the fact that he was drawn into other spheres of work—one of which was the passing of the first theological examination at Stettin—caused this seed-idea to remain dormant for some considerable time.

In October, 1834, Grassmann returned to Berlin, this time as mathematics master in a trade school. Soon afterward he applied for a better position than the one he held and his principal gave the following characterization of him: "Mr. Grassmann is a young man not lacking in attainments. It is also apparent that he has given particular attention to the elements of mathematics, and thinks with especial clearness along that line, but he seems to have had

⁵ Preface to the first edition of the 1844 *Ausdehnungslehre*.

⁶ The notion of writing $AB + BC = AC$ whether the three points A, B, C are in the same straight line or not.

⁷ Cf. J. G. Grassmann, *Raumlehre*, Part II, p. 164, and his *Trigonometrie*, p. 10.

little intercourse with people and is therefore backward in the usual forms of social life, shy, easily embarrassed and then very awkward. In the classroom all this vanishes when he does not know that he is observed. He then moves with ease, control, and certainty. In my presence, in spite of the fact that I have done all I could to give him confidence, he has not been able to become fully master of his embarrassment, which caused him much concern. My judgment of him is therefore as yet uncertain, and I cannot say whether he will be able suitably to fill the present vacancy."

As a matter of fact the vacancy was not an easy one to fill, since it had previously been held by no less a person than Jacob Steiner, the geometrician, who had been appointed to the university but retained some of the higher classes in geometry. Grassmann obtained the appointment; and as Steiner had bound himself to initiate his successor as far as possible into his own method of geometrical instruction, one would have expected interesting developments from the contact between the two men. There appears however to have been very little intimacy between them. There was a difference of thirteen years in their ages, and a wide contrast in temperament—the one self-reliant but thoroughly one-sided, the other diffident and many-sided. To these differences in personal characteristics Carl Müsebeck⁸ is inclined to attribute their small effect on each other. Victor Schlegel's⁹ view was that it was caused by the great difference in the methods employed by the two mathematicians. Whatever may have

⁸ Carl Müsebeck, article on Hermann Grassmann, No. 3, Jahrgang 6 of the *Mathematisch-Naturwissenschaftliche Blätter*, p. 1, note.

⁹ It is curious to note that V. Schlegel, who, as we shall see, was one of the first appreciators of Grassmann's work, long afterward used the methods of Grassmann's "Geometrical Analysis" to attack the problem of the minimum sum of the distances of a point from given points (*Bull. Amer. Math. Soc.*, Vol. I, 1894, p. 33) and reached a general result which reduces to Steiner's form of solution as a special case; thus illustrating the power of the method.

been the cause it is at any rate clear that Steiner's method of handling geometry had no influence whatever upon Grassmann's manner of thinking.

Several things combined to make Grassmann's stay in Berlin short. He was greatly distressed by the loss of his youngest sister, who was scarcely four years old, and this increased his inclination to religious brooding—to which he was the more inclined as he lacked suitable companionship. His eyesight also gave him some trouble, so that after a year and a quarter he gladly returned to Stettin on January 1, 1836, and became teacher in the Ottoschule.

He had, however, pleasant memories of these months in Berlin, as we can see from a letter written to his brother Robert, in which after speaking with pleasure of his return to Stettin he acknowledges the freedom and mental stimulation afforded by Berlin. At first glance this move from the capital seems a pity, since recognition of his talents might have come to him if he had stayed on. But we must remember to set against this, that he was very high-strung and energetic in mind and could be easily over-stimulated—an effect helped by the quiet life he lived—and also that a calmer atmosphere was more suitable to the long and careful development of his very original way of thought.

While still at the Ottoschule Grassmann entered for and passed the second theological examination in Stettin in July, 1839. We may note here that he was deeply attached to the study of positive theology throughout his life. After passing his theological examinations he became secretary and then president of the "Pomeranian Central Society for the Evangelization of China." And it is noteworthy in this respect that his last work was on "The Falling Away from Belief."

A few months before he submitted his essay for this last theological test, he was examined by the Berlin Scien-

tific Examination Commission in mathematics and physics. It was in connection with this that an event fraught with great consequences to his lifework happened to Grassmann; for he was set the task, by Professor Conrad of the Joachimsthal Gymnasium, of developing the theory of tides. It is uncertain whether the subject was chosen by Conrad on his own initiative or was suggested by Grassmann himself. In any case it was precisely the practical need which was best calculated to spur him on to the development of his dormant mathematical ideas. Later on he spoke¹⁰ of the necessity, in expounding the claims of a new mathematical discipline, of showing its application. And it seems clear that, faced with the difficulties and complications of Laplace's tidal theory, he was led at once to the idea of transforming analytical mechanics by the introduction of his own rudimentary analytical notions. He found to his delight that the new analysis proved a powerful simplifying tool when applied to the equations of Lagrange's *Mécanique analytique*. This initial success encouraged him to extend his method and to clothe many other conceptions such as exponentials, the angle, and the trigonometrical functions, in the form of that analysis. He was then able to simplify and render symmetrical the intricate formulas of the tidal theory. Furthermore he found that the elimination of arbitrary coordinates so effected left the ideas, their development, and their interrelations much less obscured by analytical machinery.

The thesis Grassmann sent to Berlin in April 1840 was of an unusual size;¹¹ and, in the opinion of Engel,¹² "judged by the number of new thoughts and methods contained in it, there is only one other to be compared with it—the thesis which Weierstrass submitted a year later to the Commis-

¹⁰ In the Preface to the first edition of the *Ausdehnungslehre* of 1844.

¹¹ It fills 190 pages of royal octavo in the third volume of his *Werke*.

¹² F. Engel, *loc. cit.*

sion at Münster." The two works were, however, accorded very different receptions; and it is evident that Professor Conrad had no idea of the remarkable work he had called into being. His report runs: "The test treats the theory of the tides with thoroughness and strength throughout; and he has chosen, not unhappily, a peculiar method which departs in many particulars from the theory of Laplace." It remains an evil omen for the fate of Grassmann's later work that his examination thesis should thus have failed to find recognition. It must be added that Conrad could scarcely have read the work and still less have been able to estimate it at its true value. For he received it on May 26 and returned it five days later at the oral examination—in which Grassmann fared better, being granted full recognition of his mathematical ability.

Grassmann probably realized that this thesis on tidal theory was but a first fruit of his methods and that those methods themselves were much more general and capable of immense development. This work he threw himself into with characteristic energy in the next few years. He left the Ottoschule at Michaelmas, 1842, and spent six months teaching at the Stettin Gymnasium; after which he entered the Friedrich-Wilhelm-Schule which had been founded a few years before, and of which his eldest son Justus Grassmann is now the principal.

By 1842 Grassmann had completed the main outlines of his new analytical method. He tried to make the ideas known to his own circle by lectures, in which he showed the power of the new "science of extended magnitudes" by further application to mechanics and crystallography. Desiring to expound his method by reference to well-known results he was led to the barycentric calculus of Möbius and to Poncelet. The first of these illustrations was the "Theorie der Zentralen" (Crelle's *Journal*, Vol. XXIV,

1842) in which, without using his own analysis, he made a general statement in which not only all Poncelet's results but also further important general properties of curves and surfaces are contained as special cases. Such wide generalization is characteristic of his method. In 1844 his *Ausdehnungslehre* was published, being designed as the first part of the complete work. This part, which he proposed to follow up with a second later, he called "*Die lineale Ausdehnungslehre*, a new branch of mathematics."

The fate of this book was a tragic one. It remained unread and unsold until the publisher had to get rid of the whole edition as waste paper. Not even a review was granted to it; and what criticism there was had so little basis of understanding that it led to no deeper study of the work. Gauss wrote of it, in 1844, that its tendencies partly went in the same direction in which he himself for almost half a century had wandered; but there seemed to him to be only a partial and distant resemblance in the tendency. He thought it would be necessary to familiarize oneself with the special terminology to get at the real kernel of the book. Grunert declared that he had not completely succeeded in forming a definite and clear opinion about the work. Möbius, whom Grassmann had asked for a review in some critical journal because he stood nearest to the ideas in the book, answered that this mental relationship only existed in regard to mathematics, not with reference to philosophy; and that he considered himself incapable of estimating and appreciating the philosophical element of the excellent work—which lies at the base of all mathematics. But he added that he recognized that, next to the great simplification of method, the principal gain consisted in the fact that by a more general comprehension of fundamental mathematical operations the difficulties of many analytical concepts are removed.

Without entering in detail into a discussion of the causes of this neglect of Grassmann's work¹³ we may note that its great generality, its philosophical form, and its original and technical symbolism were contributing factors which also make it very difficult to give any account of the work for the general reader.¹⁴ But the importance of the ideas hidden away in this forbidding volume may be gathered from the words written of it by Carl Müsebeck many years later: "Earlier than Riemann, Grassmann evolved manifolds of n dimensions in mathematical analysis. In a lighter and less constrained manner Grassmann arrives by his combinatory multiplication at the fundamental principles of determinant-theory, and the elementary solution of various problems of elimination. In him one finds indicated both Bellavitis's Equipollences and Hamilton's Quaternions." And yet the only recognition given by mathematicians to the ideas of Grassmann was the award to him by the Jablonowski Society at Leipsic for a prize essay¹⁵ on the "Geometrical Calculus of Leibniz" in 1846.

It must not be supposed, however, that Grassmann sat quietly down to neglect. He brought out the importance and applicability of his investigation by numerous valuable articles in Crelle's *Journal*, and later in *Mathematische Annalen* and the *Nachrichten* of the Royal Society of Science of Göttingen. Furthermore, in 1845 he published in Grunert's *Archiv*, Vol. VI, a detailed abstract¹⁶ of the *Ausdehnungslehre*, intended for mathematicians. Thirty years later Grassmann spoke to Delbrück¹⁷ with youthful ardor

¹³ See the article below on "The Neglect of the work of H. Grassmann."

¹⁴ An attempt was made to do this by Justus Grassmann in an address delivered at the opening of his school year on April 16, 1909, when the centenary of his father was being celebrated.

¹⁵ *Geometrische Analyse*, published 1847. This treatise is to some extent a substitute for the second part of the *Ausdehnungslehre* of 1844, anticipated in the preface to that work but never written.

¹⁶ Reprinted in the *Werke*, Vol. I, Part I, p. 297.

¹⁷ B. Delbrück, "Hermann Grassmann," Supplement to the *Allgemeine Zeitung*, Oct. 18, 1877.

of this period as one of happy restlessness and joy in discovery. Such joy in original work and faith in the power of his mathematical methods he always retained in spite of a succession of disappointments which would have quenched a less ardent spirit.

It is an extraordinary thing that it was not only in his mathematical work that he failed to find recognition, but also in his contributions to physics. In 1845 he published in Poggendorff's *Annalen* a statement of the mutual interaction of two electric stream lines which was re-discovered thirty-one years later by Clausius. In a school syllabus in 1854 Grassmann stated that the vowels of the human voice owe their character to the presence of certain partial tones of the mouth cavity, a view of the nature of vowel sounds which is usually ascribed to Willis and Helmholtz. Of his other purely physical work we may mention his notes on the mixing of colors and his design of a very simple but practical heliostat.¹⁸ Still he continued to hope that the value of his work would be appreciated. He had himself foreseen¹⁹ that the dislike of mathematicians for a philosophical form might deter them from considering his work, and the comments of Möbius and Grunert on this had shown his fears to be well founded. So he yielded to the often expressed wish of Möbius that he should rewrite the *Ausdehnungslehre* in a form more attractive to mathematicians. In the new work, published in 1862, he chose a more deductive method—one moreover which is not altogether suited to the subject matter, but it did succeed in bringing forward more clearly the original operations and characteristics of the *Ausdehnungslehre*. All was in vain. Neither genius nor indomitable energy could contend against so unresponsive an environment.

We must remember that Grassmann's continued output

¹⁸ A model was constructed by the Stettin Physical Society.

¹⁹ Preface to the first edition of the 1844 *Ausdehnungslehre*.

of virile original work was done in the scanty leisure of an energetic schoolmaster. He had been nominated head-teacher at the Friedrich-Wilhelm-Schule in 1847, and five years later he was appointed successor to his father at the gymnasium. There he remained for a quarter of a century. He had hoped that his mathematical writings would win for him some position in which he would have more leisure for research and be in closer contact with other scientific workers. But it must not be supposed for an instant that this lessened his intense interest in the work at hand. He wrote articles on educational subjects as well as a number of text-books for school use. Of these his *Arithmetik*, written in collaboration with his brother Robert showed a strictness in its proofs which made it a good introduction to the theory of numbers. His *Trigonometrie* has a richness of content in small space and an originality of plan not often then found in elementary hand-books.

Müsebeck has questioned some of Grassmann's pupils on his methods of teaching. They appear in the main to agree with Wandel, who says in his "Studies and Characters from Ancient and Modern Pomerania" that he was a lovable and painstaking master whose kindly instruction was sometimes too difficult for them. The lively interest he took in the independence of those he taught is shown by the fact that, according to Schlegel, he formed a society out of every three scholars in his chemistry class, the members of which had to demonstrate and lecture to the others on some substance and its combinations. The pleasant footing he established between himself and his classes may be judged from the fact that they were willing to co-operate in classwork with him when in later years he had to be taken to school in a wheeled chair. Whenever any of his old pupils speak of him they do so with the greatest admiration and respect.

It is difficult in thus giving an account of Grassmann's educational and scientific activity to avoid at the same time conveying the impression of a mere enthusiastic pedant. It does not seem that there could have been time for anything else. And yet such a view would be widely removed from the truth. For in the midst of all these exacting duties he had many social and general interests. In 1848 he took an active part in politics, expressing anti-revolutionary sympathies; he attempted to introduce a German plant-terminology into botany; and his early developed love for music found expression in organizing an orchestra of his scholars and in collecting numerous folk-songs, which he set for three voices, to be sung in his family.

We have been led, by the necessity of obtaining some idea of the actual conditions under which Grassmann worked, to speak of his later life. We must now return to the time when he first began to realize how slight a recognition was to be accorded to his mathematical writings—that is to say about the year 1852. Great as was his inner sureness of the value of the work, yet his was not the type of mind to be satisfied with a partial success. And so he took the astonishing (and almost unprecedented)²⁰ step of turning his attention to another field of knowledge altogether and quickly winning the recognition of experts. The pliability of his genius enabled him to force his way into a new subject, philology, and to produce results of outstanding merit in it.

B. Delbrück²¹ gives an interesting account of how Grassmann turned to philology. The rules of the traditional school grammar with its mass of exceptions must have been painful to his mathematical understanding, and

²⁰ The equally neglected English genius Thomas Young combined mathematical and philological ability.

²¹ B. Delbrück, *loc. cit.*

so he first planned a grammar and reading book in which scientific laws replaced the old rule-of-thumb methods wherever possible. It is natural therefore that he should next turn his attention to that sphere of language in which such laws are most easily recognizable, namely phonetics. His first attempt in the realm of comparative philology was on this subject. It was an article, published in 1859 on the influence of *v* and *j* on neighboring consonants, and on certain phenomena in connection with aspiration. Delbrück expresses the opinion that his work in this field is not distinguished either for breadth of scholarship, since he worked with few books, or for etymological depth. "But," he says, "it is the clearness of reflection which penetrates into all corners of the subject, the persistence with which the material has been so long accumulated until it became possible to reach the simplest formulation of the governing law, and the untiring nature of the mathematical abstraction which in these undertakings so clearly comes to light."

Grassmann must have quickly recognized how valuable in all researches into comparative philology a deep acquaintance with the oldest Indian languages would be, and he determined with his usual persistency to make himself at home in the hymns of the Vedas. These Sanskrit studies led him to the production of works which rendered his name famous. In 1861 he had only the first volume of the upright text and scarcely half of the Böhtlingk-Roth dictionary. Yet with these means he succeeded in mastering the extraordinary difficulties of the texts, and began his dictionary and translation of the Rig-Veda. He arranged his dictionary in an original manner so as to be able to give the meaning of each form according to the place in which it occurred. Although Delbrück credits the first volume of the dictionary with etymological value for its grammatical subdivision of the roots, yet he regards the arrange-

ment just mentioned as unphilological. It aims less at giving definite historical and philological information than at making successive attempts at explanation. As, however, the work progressed, aided by the stream of material reaching the author from the growing Roth dictionary and elsewhere, it became more philological. Still the method pursued was the same, and Grassmann completed the translation side by side with the dictionary. For long these works formed a useful tool in attacking the difficulties of the Vedas. The recognition of experts was worthily expressed by Rudolph Roth, on whose word the University of Tübingen conferred upon Grassmann the honorary degree of Doctor of Philosophy. He spoke of him as a man *qui acutissima vedicorum carminum interpretatione nomen suum reddidit illustrissimum*.

During this period of his life when he was winning fame in another sphere of work, Grassmann's mathematical writings were gradually obtaining the recognition which was their due. Toward the end of the sixties considerable attention was paid by mathematicians to higher algebra, and the quickening of thought along those lines made recognition much more likely. Hermann Hankel in his *Theorie der complexen Zahlensysteme* of 1867 was the first to call attention to Grassmann's work. Clebsch²² also shortly afterward accorded him a full measure of admiration. Grassmann²³ believed that Clebsch would have fertilized the theory of extension with far-reaching new ideas of his own if death had not cut short his promising career.

Some of the younger teachers at the Stettin Gymnasium had become pupils of Grassmann; and one of these, the mathematician Victor Schlegel, in his *System der*

²² Clebsch, *Zum Gedächtniss an Julius Plücker*, 1872.

²³ See preface to second edition of the *Ausdehnungslehre* of 1844, published in 1878.

Raumlehre (1st part 1872, 2d part 1875) made his works more accessible by a clear exposition and application of them. The best kind of approval from authorities came to him in their use of his methods in various fields; and Grassmann himself, after a long interval, again took up his mathematical labors. Of the many articles from his pen, we may mention especially that on the application of his work to mechanics,²⁴ because it was in this domain that he considered the theory of extension to be particularly successful. He expressed the desire that it might be granted to him to write a treatise on mechanics based on his principles. This was denied him. He lived, however, to see a second edition of his ill-fated *Ausdehnungslehre* of 1844 called for; and died, while it was passing through the press, on September 26, 1877, in his sixty-ninth year. To the last, in spite of great bodily suffering, he retained his vigor and enthusiasm. Five essays published in the year of his death testify to this.

It is a pleasant thing to think that he received such rich recognition before he died; though it must always remain a source of regret that he never succeeded in obtaining the position he hoped for, which would have enabled his powers to be more fully developed and his influence more widely expressed. And yet, there can be no cause for sorrow if we think of the fortitude of this strong soul, and remember the firm conviction expressed in the closing words of the introduction to the *Ausdehnungslehre* of 1862, that his mathematical ideas would some day arise again, though perhaps in a new form, and become part of living thought. To some extent that conviction has proved a justifiable one. The publication of his *Collected Works* was suggested by Professor Klein of Göttingen. After ob-

²⁴ "Die Mechanik nach den Principien der *Ausdehnungslehre*," *Math. Annalen*, Vol. XII, 1877.

taining the consent of Grassmann's relatives he laid the matter before the Royal Saxon Academy of Sciences in October, 1892. A committee was formed and F. Engel made chief editor. The first part of the first volume appeared in 1894.

Since then there have been many works on the calculus of extension, but it can scarcely be held that they have done more than make a beginning of the development of the suggestions in Grassmann's work. What has been done has been mainly in the domains of spatial theory and higher algebra; mechanics remains still burdened with traditional coordinate systems. This is the more remarkable since the principle of relativity, with its demand for a generalized dynamics of which ordinary dynamics is a special case, offers such a promising field of application.

There is usually, in the sphere of thought, a rational explanation of apparently irrational facts. A minute influence translated into action by the mass of thinking men may give rise to the spirit of their age; and thus its effects, and the negative effects may be just as great as the positive, carried forward in ever-increasing circles to distant generations. So it has been with whatever lies at the base of the neglect of Hermann Grassmann. There has been bequeathed to us something like an unreasoning distaste for his and similar analytical methods, from which has arisen the need for a definite effort to break the spell of the past. The formation of an "International Association for Promoting the Study of Quaternions and Allied Systems of Mathematics" took its origin from such a need.²⁵ It may therefore be that a just estimate both of the value and limitations of Grassmann's work will only come by the application of a critical method of wider scope than those of his

²⁵ P. Molenbrock and Shunkichi Kimura, letter to *Nature*, Oct. 3, 1895.

own period. Indications are indeed not wanting that in the modern theory of transformation-groups²⁶ lies the criterion for a final judgment.

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²⁶ Lie and Engel, *Theorie der Transformationsgruppen*, Vol. II, p. 748; M. Abraham and P. Langevin, "Notions géométriques fondamentales," *Encyc. des sciences mathématiques*, Tome IV, Vol. 5, p. 2.

²⁷ I wish to thank Miss Vinvela Cummin and Mr. R. E. Roper for help in the translation of materials for this sketch.